Recebido: 06-03-2023 | Aprovado: 03-04-2023 | DOI: https://doi.org/10.23882/rmd.23142

Improved Scoring of Satisfaction with Life Scale: Scoring of SWLS

Pontuação aprimorada de satisfação com a escala de vida: Pontuação de SWLS

Satyendra Nath Chakrabartty, Indian Statistical Institute, Indian Maritime University, Indian Ports Association (chakrabarttysatyendra3139@gmail.com)

Abstract: The Satisfaction with Life Scale (SWLS), a 5-item instrument with 7 response-categories per item, measures cognitive judgments of satisfaction with one's life using summative scoring where higher score implies higher life satisfaction. Discrete scores of SWLS using Likert-type items fail to satisfy equidistant property and do not consider distributions of item scores or scale scores. The paper gives an assumption-free method to convert item scores of SWLS to continuous scores following normal distribution and scale score is taken as sum of such normally distributed item scores. The proposed scores avoiding tied-scores facilitate better admissibility of arithmetic aggregation, meaningful comparisons, ranking, classification of individuals, assessment of progress/deterioration across time and undertaking parametric statistical analysis. In addition, it helps to assess changes in successive time-periods for an individual or a group of individuals, drawing of progress-paths and test of significance of progress or deterioration. Normality of proposed scores facilitates finding equivalent scores of several measures of life satisfactions with different scale formats and different score-ranges. Empirical illustration is given with hypothetical data. The proposed method is recommended to enhance the utilization of the SWLS for better measurement of satisfaction with life along with meaningful comparisons and inferences.

Keywords: Assessment of progress, Equidistant scores, Equivalent scores, Likert items, Normal distribution, Satisfaction with Life Scale.

Resumo: A Escala de Satisfação com a Vida (SWLS), um instrumento de 5 itens com 7 categorias de resposta por item, mede os julgamentos cognitivos de satisfação com a vida, utilizando a pontuação sumativa onde uma pontuação mais alta implica uma maior satisfação com a vida. Pontuações discretas de SWLS usando itens do tipo Likert- não satisfazem a propriedade equidistante e não consideram distribuições de pontuações de itens ou pontuações de escalas. O artigo dá um método sem pressupostos para converter as pontuações dos itens de SWLS em pontuações contínuas seguindo a distribuição normal e a pontuação da escala é tomada como soma de tais pontuações de itens normalmente

distribuídas. As pontuações propostas, evitando pontuações empatadas, facilitam uma melhor admissibilidade da agregação aritmética, comparações significativas, classificação, classificação de indivíduos, avaliação do progresso/deterioração ao longo do tempo e realização de análise estatística paramétrica. Além disso, ajuda a avaliar alterações em sucessivos períodos de tempo para um indivíduo ou grupo de indivíduos, desenho de percursos de progresso e teste de significância do progresso ou deterioração. A normalidade das pontuações propostas facilita a obtenção de pontuações equivalentes de várias medidas de satisfação de vida com diferentes formatos de escala e diferentes escalas de pontuação. A ilustração empírica é dada com dados hipotéticos. O método proposto é recomendado para melhorar a utilização do SWLS para uma melhor medição da satisfação com a vida, juntamente com comparações e inferências significativas.

Palavras-chave: Avaliação do progresso; pontuações equidistantes; Pontuações equivalentes; Curtir itens; Distribuição normal; Escala de Satisfação com a Vida.

1. Introduction:

The concept of life satisfaction, a key part of subjective well-being is widely-used in psychology literature in general and positive psychology in particular. Broadly speaking, two aspects of subjective well-being are (i) affective, which is further divided into pleasant affect and unpleasant affect (Diener and Emmons, 1984), and (ii) cognitive, referred to as life satisfaction (Andrews and Withey, 1976). Different scales have been evolved to measure life satisfaction, involving a number of factors and variables. Illustrative list of scale includes Satisfaction with Life Scale (Diener et al. 1985); Student's Life Satisfaction Scale (Huebner, 1991); Life Satisfaction Measure based on Judgment Theory (Meadow., 1992); Temporal Satisfaction with Life Scale (Pavot et al., 1998); Brief Life Satisfaction Scales (Lubin and Van Whitlock, 2004); Riverside Life Satisfaction Scale (Margolis et al., 2018), etc.

The most common scale is the "Satisfaction with Life Scale" (SWLS) which measures satisfaction of people's lives as a whole and not specific satisfaction area like health, energy, finances, etc. but allows subjects to integrate and weigh the domains in whatever way they choose. It may be noted that Life satisfaction is the extent of overall quality of his/her own life-as-a-whole which is different from aspect-wise judgments like satisfaction with one's work or marriage. Thus, it measures only the cognitive component of Subjective well-being (SWB). The

affective components and SWLS scores cannot be used as direct measures of emotional well-being, which implies that SWLS is a judgmental process, where respondents try to assess quality of their lives on the basis of their own sets of criteria (Shin and Johnson, 1978). However, it is not clear what standards or conditions based on which people assess their life satisfaction (Tate, 2010). Thus, SWLS is a reflective measurement model, and not a formative model. SWLS score of an individual might vary across time, and items may vary in terms of susceptible to change (Pavot and Diener, 1993)

SWLS consists of five number of 7-point Likert items, where 1: Strongly Disagree; 2: Disagree; 3: Slightly Disagree; 4: Neither Agree or Disagree; 5: Slightly Agree; 6: Agree and 7: Strongly Agree. The five items are:

1. In most ways my life is close to my ideal.
2. The conditions of my life are excellent.
3. I am satisfied with my life.
4. So far I have gotten the important things I want in life.
5. If I could live my life over, I would change almost nothing

Score of an individual is taken as summative score of the five items which ranges between 5 to 35. Higher score implies higher life satisfaction. People with higher sores in SWLS are more compatible with and productive in society (Avcu, 2021) who found that SWLS items did not show gender-bias i.e. SWLS items functioned similarly for men and women. The fifth item has been questioned due to its focus on desire to change rather than their current sense of life satisfaction (Pavot and Diener, 1993).

Meaningful comparisons of mean SWLS-scores across gender, culture, age-groups, etc. may not always be valid (Emerson et al. 2017) since ability of SWLS to measure its underlying constructs equivalently across different subgroups is usually assumed and not tested (Kern et al.2016). Comparisons of groups presume that SWLS scores are valid across various sub-groups i.e. measurement invariance (MI)

which demonstrates that a construct has the same meaning to those sub-groups or across repeated measurements. For a given factor score of a subject, MI indicates his/her observed score is independent of his/her group membership (Lubke et al. 2003). Oishi, (2006) examined MI of SWLS between students of USA and China using multi-group structural equation modeling, multiple indicators multiple causes, and item response theory (IRT) and found that Items 4 and 5 were non-invariant. Similarly, using latent-class analysis for testing MI of SWLS between USA and China, Eid et al. (2003) found that MI did not hold. There is no agreed method of testing MI and different methods used by researchers may give different results. Jang *et al*. (2017) used three different methods to test MI and found that scalar MI did not hold across countries. Strict MI satisfying the following four conditions across groups are rarely used in applied context (Van De Schoot et al. 2015):

1) *Equal form*: The number of factors and same factor-indicator relationships

2) *Equal loadings*: Equal Factor loadings.

3) *Equal intercepts*: When observed scores as dependent variables and each factor scores are taken as independent variable in regression equations

4) *Equal residual variances*:

Welzel et al. (2022) described limitations of MI tests as a tool of measurement validation which works well for constructs with low between-group variance but not the constructs having high between-group variance. Invariance is a guiding principle and an ideal for model-based measurement theory but not a property of the SWLS. One possible way to avoid the problem is to transfer SWLS scores to follow normal distributions, parameters of which could be different for different subgroups.

The paper gives an assumption-free method to convert ordinal SWLS score to continuous score following normal distribution and to facilitate meaningful comparisons, ranking, classification of individuals, assessment of progress/deterioration across time and undertaking parametric statistical analysis. In addition, such transformations also help to find equivalent score of SWLS and another scale, keeping in mind prediction is different from finding equivalent scores.

2. Literature survey:

Summative SWLS score inter alia assumes:

- Equal importance to the items despite different values of item-total correlations, item reliabilities, factor loadings. Pavot and Diener (1993) cited four studies where factor loadings and item-total correlations of each SWLS item varied, and the 5th item had lowest factor loadings and lowest item-total correlations. Item-wise coefficient of variation (CV) = $\frac{Standard\ deviation\ (SD) of\ item}{Item\ mean}$ where lower value reflects consistency were computed by Pavot et al. (1991) for SWLS items and found maximum CV of 43.76% for Item-5, followed by 36.84% for Item-4, 32.07% for Item-2, 31.21% for Item-1 and 29.06% for Item-3.

- Response-categories are taken as equidistant, i.e. distance between (say) "Disagree" and "Slightly Disagree" is same as distance between "Agree" and "Strongly Agree". However, distances between successive levels of an item are different and unknown which imply arithmetic aggregations are not meaningful (Bastien, et al. 2001).

- Response-categories are assumed to convey the same to the respondents, which may not be true. Individuals differ on their perceptions about Ideal life, Conditions of excellent life, Satisfied with life, Important things in life, etc. and how frequently an action is to take place for regarding it as *most ways*. Participants may find it difficult to distinguish between adjacent category levels (Avcu, 2021).

- Suggested cut-off points for SWLS for benchmarking by Pavot and Diener (2013) are:

- 31 - 35 Extremely satisfied [chosen levels: 6 to 7 for each item]
- 26 - 30 Satisfied [chosen levels are 5 to 6]
- 21 - 25 Slightly satisfied [chosen levels are 4 to 5]
- 20 Neutral [level 4 is chosen for each item]
- 15 - 19 Slightly dissatisfied [chosen levels are 3 to 4]
- 10 - 14 Dissatisfied [chosen levels are 2 to 3]
- 5 - 9 Extremely dissatisfied [chosen levels are 1 to 2]

One limitation of such classification is that it fails to distinguish subjects with equal individual score i.e. individuals with tied score. For example, consider score pattern of two individuals as 1-4-5-6-7 and 7-6-5-4-1. Here, summative score of each individual is 23 i.e. both are slightly satisfied, as per the above classification. But, overall satisfaction level of the 1st individual (with high scores in 4th and 5th items) appears to be more than the 2nd individual (with low scores in 4th and 5th items).

Efficiency of classification in terms of ratio of within group variance (small value for a good classification) and between group variance (high value for a good classification) need to be undertaken to assess goodness of classification, say by Wilks' lambda which assumes normally distributed variables. Number of class need not be equal to number of response-categories. If the objective is to classify the subjects in two mutually exclusive classes, "Dissatisfied" and "Satisfied" then better is to adopt parametric Receiver Operating Characteristic (ROC) curve analysis and decide cut-off score (C_0) of SWLS in a specific population such that persons with scores less than C_0 are dissatisfied and those with scores exceeding C_0 are satisfied. Such analysis presumes normally distributed scores of SWLS.

Factors of life satisfaction have been measured using variables like health-status, employment, economic status, and level of activity (Diener and Chan, 2011). Three major factors which influence SWL are social relationship, performance with respect to goals derived from one's values and personal satisfaction (with the self, religious or spiritual life, learning and growth, and leisure) (Pavot and Diener, 2013).

Subjective responses of individuals to SWLS could be based on his/her past life (in retrospect), or future/expected life (in prospect) (future life), or the present (current life). Thus, stability of judgments of the respondents in terms of SWLS scores may vary with time. This can be reflected by different values of correlation between repeated administrations of SWLS on the same cohort across time. Factors like age, education level, cognitive level, personal capabilities and social relations etc. of the subjects also affect stability of SWLS scores with time. Ehrhardt et al (2000) showed that the log-graph of correlation of life-satisfaction variable at base period t_0 and j-th period t_j i.e. $logr_{t_0 t_j}$ will be convex initially and asymptotic as j increases.

The SWLS is a valid and reliable measure of life satisfaction and has been used worldwide in clinical and non-clinical populations across age-groups, occupations, genders and patients suffering from chronic illnesses like arthritis (Laranjeira,2009), mental illness (Meyer *et al.* 2004), systemic lupus erythematosus (Kulczycka, *et al,* 2010), Parkinson's disease (Lucas-Carrasco *et al*., 2014), etc.

Factor structure of SWLS were attempted using Principal Component Analysis (PCA), Factor analysis (FA), where a single factor resulted explaining over 60% of the variance of the scale implying one-dimensional structure of the scale (Clench-Aas *et al*., 2011; Pavot and Diener, 1993;Vázquez *et al.,* 2013). However, factor loading of the fifth item was low in comparison to the other items (Vázquez *et al*., 2013).

Reliability of SWLS in terms of Cronbach Alpha is common. For example, alpha was 0.82(Vera-Villarroel *et al*. 2012), 0.84 (Galanakis *et al.* 2017), 0.85 (van Beuningen, 2012), 0.88 **(**Vazquez, *et al*. 2013). However, high value of alpha is not a sufficient condition for measuring homogeneity or unidimensionality of the scale (Cortina, 1993). Too high value of alpha may suggest that some items could be redundant in the sense that these are measuring the same question but in a different form. One way is to see effect of deletion of an item (say the item with minimum item-total correlation) on alpha. However, studies reporting such deletions of item and the resulting values of alpha are rare. López-Ortega *et al.* (2016) used Cronbach alpha and inter-item correlations to find reliability of SWLS and conducted exploratory factor analysis (EFA) which resulted in a single factor with eigenvalue of 2.71 and explaining 54.2% of the variance implying coherence to life satisfaction. Each such technique requires meaningful admissibility of arithmetic aggregation.

SWLS was correlated well with Subjective Happiness Scale (SHS) with four items rated on a 7-point Likert scale (Karakasidou *et al*. 2016). Similar high correlations of SWLS were observed with the Meaning of Life, a 10 item measure with 7-point scales (Steger *et al*, 2006), Life Satisfaction Index – 20 items, each in 3-point scale (Abdallah, 1998; Pavot *et al.,* 1991), Hope scale - 12 number of items each in 8-point scale (Bailey and Synder, 2007), etc. SWLS was negatively correlated with measures of stress (- 0.39), depression (- 0.34) and anxiety (- 0.39) (Mahmoud *et al.* 2012).

High correlation prompts to predict SWLS score with knowledge of score of other scale (say) SHS or vice versa. But there is no cause-and-effect relationship between SHS and SWLS. Correlations may not be suitable for analysis of agreement of ordinal scores emerging from two scales since correlation is different from agreements. The pertinent issue is to find equivalent score of SWLS for a given SHS score, keeping in mind prediction is different from finding equivalence (Livingstone, 2004). Using the probability density function of SHS (*f(x)*) and SWLS (*g(y)*), one can find equivalence score combinations $\{SWLS_0, SHS_0\}$ so that area under the *f(x)* up to SHS_0 = the area under the *g(y)* up to $SWLS_0$, irrespective of scale format i.e. number of items and number of response-categories.

High correlation between X and Y where X denotes SWLS score and Y denotes score of the other scale may indicate SWLS has high construct validity. But question may arise whether the correlation r_{XY} indicates validity of X or Y? In addition, high value of r_{XY} could be due to similarity of the latent variable(s) being measured by X and Y (validity) or due to similarity in distribution of X and Y or due to nature of data and score-range of one or both the variables. For example, let X follows *N (0, 1)* and Y is the ordinate of *N (0, 1)* i.e. $Y = \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}X^2}$ Clearly, X and Y are not linear. Here, r_{XY}= - 0.93302 if $0 \leq X \leq 3.9$ (Case 1) and r_{XY}= 0.00036 if $-3.9 \leq X \leq 3.9$ (Case 2).Interpretation of r_{XY} from Case 1 is X and Y are highly correlated but correlation is negative i.e. increase of one unit in X will result in decrease of Y and vice versa. However, interpretation of r_{XY} from the Case 2 will be just the reverse. The example shows that truncated score-range can affect $r_{XY}$and high r_{XY}does not mean linearity between two variables and justification for fitting regression lines of Y on X or X on Y needs to be given by testing error variance is insignificant.

Distribution of SWLS score and item-wise SWLS are not known. Galanakis *et al.* (2017) **c**oncluded normal distribution of each SWLS item based on item mean lying between 4.07 to 4.80 and $2.22 \leq \text{Item variance} \leq 3.56$. This can be questioned since narrow range of mean and SD does not guarantee normal probability distribution function. Inter-item correlations ranging between0.44 to 0.62; tend to indicate unidimensionality of SWLS. However, this needs to be confirmed by PCA or FA.

Psychometric properties of SWLS were empirically investigated by Avcu (2021) using IRT approach, primarily to provide results which are independent of sample. Vittersø *et al*. (2005) fitted a mixed Rasch model to study cultural differences on SWLS items and reiterated need to use care in analyzing survey data across cultures. Rasch analysis of SWLS showed that the scale was insensitive at high score levels of life satisfaction (Schutte *et al*. 2019). However, the complex IRT technique involves strong assumptions like equal discriminating value for the items, unidimensionality of scale, local independence of the items, curvilinear relationship between item score and constructs score which may not be good approximations of the reality of testing (Livingstone, 2004). Moreover, transforming scale scores to follow normal distribution can facilitate estimation of population parameters and drawing conclusions accordingly.

3. Proposed method:

It is a multi-stage method to convert ordinal SWLS scores to continuous, monotonic, equidistant scores followed by standardization and further linear transformation to ensure $1 \leq$ Proposed score $\leq 100,$ proposed by Chakrabartty, (2019)

Let X_{ij} denote the response endorsed by of the *i*-th person in the *j*-th item, for $i = 1,2, \ldots, n$ and $j = 1, 2, \ldots \ldots, 5$. X_{ij} takes discrete value 1, 2, 3, 4, 5, 6 and 7 for a SWLS item.

3.1 Equidistant score:

Assign weights $W_{ij}'s$ to different levels of different items so that $W_{ij} > 0, \sum_{j=1}^{7} W_{ij} = 1$ and take item score of an individual as weighted sum satisfying equidistant condition i.e. $W_1,\ 2W_2,$

$3W_3, 4W_4, 5W_5, 6W_6$ and $7W_7$ forms an arithmetic progression with a positive common difference. One way to find such weights W_{ij} is given below:

Find maximum (f_{max}) and minimum frequency (f_{min}) of levels of each item. For the *i*-th item, put initial weights as $W_{i1} = \omega_{i1} = \frac{f_{min}}{n}$. Find the common difference $\beta = \frac{7(f_{max} - f_{min})}{6n}$

Define$W_{i2} = \frac{\omega_{i1} + \beta}{2}; W_{i3} = \frac{\omega_{i1} + 2\beta}{3}; W_{i4} = \frac{\omega_{i1} + 3\beta}{4}$, $W_{i5} = \frac{\omega_{i1} + 4\beta}{5}$, $W_{i6} = \frac{\omega_{i1} + 5\beta}{6}$ and $W_{i7} = \frac{\omega_{i1} + 6\beta}{7}$. Choose final weights $W_{ij(Final)} = \frac{W_{ij}}{\sum_{j=1}^{7} W_j}$ so that $\sum W_{ij(Final)} = 1$

The procedure can be well applied for *k*-point scale for *k* = 3, 4, 5, 6,…. and so on. Here, score of *i*-th person is taken as expected values and thus, the score (E_i) is continues, monotonic (choice of *(j*+1)-th response category instead of the *j*-th category of an item will increase *E*-score of an individual) and equidistant property is satisfy since $j.W_j - (j-1)W_{j-1} =$ constant, for an item.

3.2 Normality:

Standardize the equidistant scores (*E*) to follow $N(0,1)$ using$Z = \frac{E - \bar{E}}{SD(E)}$. To avoid negative values, transform *Z* -scores by a linear function to have a desired range. Proposed score (*P*) in the range of [1, 100] can be obtained from Z by $P = \frac{(100-1)(Z_{ij} - Min(Z_{ij})}{Max(Z_{ij}) - Min(Z_{ij})} + 1$ (1)

Sum of transformed item scores i.e. $P = \sum_{i=1}^{n} P_i$ is taken as score of scale containing *n*-items, following Normal distribution. Note that if item scoresP_1, P_2,....., P_n are not independent and $P_i \sim N(\mu_i, \sigma_i^2)$, then scale score $P = \sum_{i=1}^{n} P_i \sim Normal$ with mean $\sum_{i=1}^{n} \mu_i$ and variance = $[\sum \sigma_i^2 + 2\sum_{i \neq j} Cov(P_i, P_j)]$

3.3 Responsiveness:

Responsiveness of the scale or ability to measure changes with time can be assessed by $\frac{P_{i_{(t+1)}}}{P_{i_t}}$ or by $\frac{P_{i_{(t+1)}} - P_{i_t}}{P_{i_t}}$ for an individual, in successive time periods. For a group of individuals, change is given by $\frac{\overline{P_{\iota_{(t+1)}}} - \overline{P_{\iota_t}}}{\overline{P_{\iota_t}}}$ or percentage change by $\frac{\overline{P_{\iota_{(t+1)}}} - \overline{P_{\iota_t}}}{\overline{P_{\iota_t}}} * 100$. Percentage change is positive if $\overline{P_{\iota_{(t+1)}}} > \overline{P_{\iota_t}}$ implying progress of the group in *(t+1)*-th period over *t*-th period. $\overline{P_{\iota_{(t+1)}}} < \overline{P_{\iota_t}}$ indicates deterioration and percentage deterioration is $\left| \frac{\overline{P_{\iota_{(t+1)}}} - \overline{P_{\iota_t}}}{\overline{P_{\iota_t}}} \right| *$ 100. Quantification of changes in life satisfaction i.e. progress/decline of a patient or a group of patients is vital for clinical samples to assess effect of treatment plan and make corrective decisions accordingly. For non-clinical samples, changes in life satisfaction is important with changes in major events of life like new employment, changed financial status, marriage/divorce, etc. Significance of progress/deterioration can be tested statistically since ratio of two normally distributed variable follows χ^2 distribution.

The curve showing P_{i_t} against time-periods *t* will give the progress-path of the *i*-th individual over time. A steadily increasing progress-path for a patient is highly desirable, but not feasible due to unstable nature of satisfaction with life. In reality, the progress-path could be zigzag showing progress and deteriorations at different time points. Comparison of progress-paths of patients or group of patients can help to draw important inferences over time. The curve showing P_{i_t} against time-periods *t* is simple to find and is an alternative to log-graph of correlation proposed by Ehrhardt et al (2000).

3.4 Integration of Scales:

Situations often demand use of multiple measures of patient-reported life satisfactions like SWLS, Subjective Happiness Scale, Meaning of Life, Life Satisfaction Index, Hope scale, etc. where the scales have different scale formats, different score-ranges and more importantly different distribution of scores and

degree of error variances. Such comparisons are challenging to decide equivalent cut-off scores for the classes emerging from different scales i.e. to find one-to-one correspondence among the equivalent score of various scales, where the scales have been administered to a common sample. The situation is different from predicting score of Scale 2 by regressing score of Scale 2 on say Scale 1.

A solution to find equivalent score combinations of two scales was proposed by Chakrabartty (2021) avoiding the problems of linear equating and percentile equating. Here, raw scores of Scale 1 and Scale 2 are converted to proposed scores for each scale separately such that $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$. Let *f(x)* be the normal probability density function of Scale 1 (X) and *g(y)* be the normal probability density function of Scale 2 (Y). A score of Y_0 in Scale 2 is equivalent to a score of X_0 and vice versa if

$$\int_{-\infty}^{X_0} f(X)dx = \int_{-\infty}^{Y_0} g(Y)dy \qquad (2)$$

That is, area under $f(X)$ up to X_0 = area under $g(Y)$ up to Y_0. For a given value of X_0, the equivalent score of Y_0 can be found by solving (3) using the Standard Normal Probability table. For example, if $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$ and a cut-off score in Scale 1 is X_0 is known, then the equation (3) can be written as

$$\int_{-\infty}^{X_0} \frac{1}{\sigma_1(\sqrt{2\pi})} e^{-\frac{(X-\mu_1)^2}{2(\sigma_1^2)}} dx = \int_{-\infty}^{Y_0} \frac{1}{\sigma_2(\sqrt{2\pi})} e^{-\frac{(Y-\mu_2)^2}{2(\sigma_2^2)}} dy \qquad (3)$$

Z-value corresponding to X_0 is $\frac{X_0-\mu_1}{\sigma_1}$ where Z $\sim N\,(0,\,1)$.

So, LHS of equation (4) becomes $\int_{-\infty}^{X_0} \frac{1}{\sigma_1(\sqrt{2\pi})} e^{-\frac{(X-\mu_1)^2}{2(\sigma_1^2)}} dx = \int_{-\infty}^{\frac{X_0-\mu_1}{\sigma_1}} Z\,dz = \beta$ (say) which can be found from the Standard normal probability table. Using the table, one can also find value of Y_0 such that $\int_{-\infty}^{\frac{Y_0-\mu_2}{\sigma_2}} Z\,dz = \beta$ where X_0 and Y_0 are equivalent ($X_0 \Leftrightarrow Y_0$). The procedure may be extended to find equivalent scores of three or more scales, even if they have different scale formats and different range of scores. Equivalent score combinations will have almost perfect correlation.

Alternate ways to find equivalent scores avoiding solution of integral equation (3) could be to convert item-wise *E*-scores to standardized scores and further transform to a normal distribution with proposed mean and variance say $N\,(50,\ 10^2)$ and drawing cumulative frequency curve (ogive) for each scale.

4. Illustration:

Illustration is given with hypothetical data involving 100 persons. Table 1 given below indicates calculation of weights for a scale with 5 items, each in a 7-point scale to get equidistant scores along with Mean and SD of raw scores, equidistant scores and *P*-score following normal.

Table 1: Weights for equidistant scores and Mean, SD of raw scores, equidistant scores and *P*-score

Description		Item 1	Item 2	Item 3	Item 4	Item 5	Total
Frequency	Max	21(L-5)	29 (L-4)	28 (L- 4)	27 (L-6)	20 (L – 1)	
	Min	8 (L -3)	7 (L - 3)	5 (L-1)	6 (L- 2)	8 (L- 4)	
Final weights	Level 1	0.09134	0.053327	0.038103	0.04857	0.097037	
	Level 2	0.132253	0.124429	0.121295	0.12345	0.133426	
	Level 3	0.145891	0.148129	0.149026	0.148409	0.145555	
	Level 4	0.15271	0.15998	0.162891	0.160889	0.15162	
	Level 5	0.156801	0.16709	0.17121	0.168377	0.155259	
	Level 6	0.159528	0.17187	0.176756	0.173369	0.157685	
	Level 7	0.161477	0.175216	0.180718	0.176935	0.159418	
	Total	1.0	1.0	1.0	1.0	1.0	
Raw scores	Mean	4.45	4.31	4.08	4.88	4.01	21.73
(*X*)	SD	1.871503	1.824054	1.580692	1.924221	2.217902	4.240755
	CV in %	42.056	42.321	38.742	39.431	55.309	19.545
Equidistant	Mean	0.691954	0.700533	0.667923	0.818087	0.608179	3.486676
Score (*E*)	SD	0.328293	0.356659	0.323231	0.381629	0.376632	0.790112
P-Score	Mean	51.18254	55.61505	51.81997	65.02	50.66501	274.3026
	SD	27.42968	30.09685	26.08143	31.749643	36.59538	68.81225
	CV in %	53.592	54.116	50.331	48.831	72.230	25.986

Legend: L (K) indicates K-th level for K= 1, 2, 3, 4, 5, 6, 7

4.1 Observations:

Major observations emerging from the table are:

- Weighted sum with different weights to different levels of different items converted ordinal raw scores of items (*X*) to continuous, monotonic and equidistant scores (*E*). Here, common difference i.e. $7W_7 - 6W_6 = 6W_6 - 5W_5 = 5W_5 - 4W_4 = 4W_4 - 3W_3 = 3W_3 - 2W_2 = 2W_2 - W_1$ for Item 1, 2, 3, 4 and 5 was respectively 0.173166, 0.195531, 0.204487, 0.198329 and 0.169815
- *E*-scores reduced mean and SD of an item and the scale. However, *E*-scores may not follow Normal.
- Item-wise *E*-scores and *P*-scores avoid equal importance to items and levels and ensure better admissibility of addition and arithmetic aggregation
- The fifth item had maximum CV= 72.23 % for *P*-score and also for raw score (CV = 55.309 %) indicating maximum inconsistency for the fifth item. This is in line with observation made by Pavot & Diener, (1993)
- Distribution of *P*-score of each item is given below:
  $P_1 \sim N(51.1825, 27.4297^2)$;
  $P_2 \sim N(55.61505, 30.0968^2)$;
  $P_3 \sim N(51.81997, 26.0814^2)$;
  $P_4 \sim N(65.02, 31.7496^2)$;
  $P_5 \sim N(50.6650, 36.5954^2)$;
  P-scores of the scale as sum of item-wise *P*-scores also follows $N(274.3026, 68.81225^2)$

Normality of proposed score helps in estimation of population mean (μ), variance (σ^2), confidence interval of μ, parametric analysis including testing of statistical hypothesis of equality of mean, variance across time and space. Population estimates of item variances (σ_i^2) and test variance (σ_T^2) help to find Cronbach alpha at population level by

$$\hat{\alpha} = \left(\frac{n}{n-1}\right)\left(1 - \frac{\sum_{i=1}^{5}\sigma_i^2}{\sigma_T^2}\right) \quad (4)$$

4.2 Tied scores:

Summative scoring of SWLS resulted in large number of tied scores since usual summative individual scores do not consider pattern of responses to Item–Level combinations. However, the *P*-score with five decimal places had no tied score. Illustrative number of tied scores resulting from raw scores (X) is shown in Table 2.

Table 2: Number of tied scores for raw scores

Score	Tie length	Score	Tie length	Score	Tie length
13	3	19	4	24	5
15	4	20	6	25	9
16	3	21	15	26	5
17	4	22	7	27	7
18	4	23	15	29	3

Here, 15 persons got score of 21. Maximum tie length of 15 is observed for $X = 21$ and $X = 23$. Illustrative example of each of seven persons with $X = 27$ and corresponding *P*-scores are given in Table 3.

Table 3: Proposed scores of seven persons who got raw score of 27

Sl.no.	Scores	Item 1	Item 2	Item 3	Item 4	Item 5	Total
1	Raw score (X)	5	5	4	6	7	27
	Proposed Score (*P*)	58.8736	67.00004	50.49997	83.49999	99.9999	359.8736
2	Raw score (X)	4	4	6	6	7	27
	Proposed Score (*P*)	44.40517	50.50005	83.50	83.49999	99.9999	361.9052
3	Raw score (X)	3	7	4	6	7	27
	Proposed Score (*P*)	29.93673	100.00	50.49997	83.49999	99.9999	363.9367
4	Raw score (X)	6	5	4	6	6	27
	Proposed Score (*P*)	73.34204	67.00004	50.49997	83.49999	83.50	357.842
5	Raw score (X)	4	4	5	7	7	27
	Proposed Score (*P*)	44.40517	50.50005	66.99998	99.99997	99.9999	361.9052
6	Raw score (X)	4	6	5	7	5	27
	Proposed Score (*P*)	44.40517	83.50002	66.99998	99.99997	67.00	361.90517
7	Raw score (X)	6	7	4	6	4	27
	Proposed Score (*P*)	73.34204	100.00	50.49997	83.49999	50.50001	357.84202

Observations:

- *X* –scores failed to discriminate persons with same individual score.
- *P*-scores enable to provide unique ranks to the individuals and may increase discriminating value of the scale.

5. Limitations

Method to find *E*-scores fails if frequency of a response-category of an item is zero. This could be taken as fixed zero point of converting SLWS scores to *E*-scores by weighted sum.

No missing data were considered. Estimations of missing data are beyond the scope of the current paper.

6. Discussion and conclusions:

The proposed method converts ordinal item-wise scores of SWLS to continuous, monotonic scale following normal distribution and score of an individual in an item lies between 1 to 100. Sum of such normally distributed item-wise scores is taken as the scale score following normal distribution, parameters of which can be estimated from the data.

Benefits of the proposed methods are:

- Avoids the problems of usual summative scores of Likert items
- Ensures better admissibility of arithmetic average
- Joint distribution of sum of item scores is Normal where parameters can be estimated from data.
- Avoids problems of evaluation of measurement invariance (MI) for which there is no agreed method. Distribution of transformed SWLS scores for different subgroups will be normal with different parameters.
- Avoids problems to assess model fit
- Facilitates parametric analysis like PCA, FA, ROC including testing of statistical hypothesis of equality of mean, variance for meaningful comparisons across time and space.
- Estimation of population mean (μ), variance (σ^2), confidence interval of μ
- Estimation of Cronbach alpha at population level
- Avoidance of tied scores and providing unique ranks to the individuals and may increase discriminating value of the scale.
- Assessment of responsiveness of the scale or ability to measure changes for an individual or a group of individuals and drawing of progress-paths
- Statistical test of significance of progress/deterioration
- Finding equivalent scores of several measures of life satisfactions with different scale formats and different score-ranges.
- The curve showing P_{i_t} against time-periods t is simple to find and is an alternative to log-graph of correlation proposed by Ehrhardt et al (2000*)* requiring cohort study for long longitudinal data with risks of familiarity and experience of the respondents in answering items of SWLS.

Considering theoretical advantages, the proposed scoring method is recommended for more meaningful comparisons and inferences. Future studies with longitudinal data can be undertaken to find sensitivity of the proposed score over time with emphasis on progression of disease and to different therapeutic interventions in clinical samples, and comparison of progress path and log-graph of correlation in non-clinical sapmel.

Declarations

Acknowledgement: Nil

CRediT statement: The single author is involved in Conceptualization, Methodology, Writing- Original draft preparation, Writing- Reviewing and Editing.

Sources of Funding: No funds, grants, or other support was received

Financial or non-financial interests: Nil

Availability of data and material: Nil. The article did not use any dataset

Approval of Ethics Committee: Not required for this methodological paper

Informed consent: No data collected from individuals

References:

Abdallah, T. (1998). The Satisfaction with Life Scale (SWLS): Psychometric properties in an Arabic-speaking sample. *International journal of Adolescence and Youth*, 7(2), 113-119. https://doi.org/10.1080/02673843.1998.9747816

Andrews, F. M., & Withey, S. B. (1976). *Social indicators of well-being America's perception of life quality.* New York: Plenum Press.

Avcu, A. (2021). Item response theory-based psychometric investigation of SWLS for university students. *International Journal of Psychology and Educational Studies, 8*(2), 27-37. https://doi.org/10.52380/ijpes.2021.8.2.265

Bailey, T. C., & Snyder, C. R. (2007). Satisfaction with Life and Hope: A Look at Age and Marital Status. *The Psychological Record, 57*, 233–240. https://doi.org/10.1007/BF03395574

Bastien, C. H., Vallieres, A., & Morin, C. M. (2001). Validation of the Insomnia Severity Index as an outcome measure for insomnia research. *Sleep Medicine*; 2(4), 297–307

Chakrabartty, S. N. (2021). Integration of various scales for Measurement of Insomnia, *Research Methods in Medicine & Health Sciences*; Vol. 2(3), 102-111. https://doi.org/10.1177/26320843211010044

Chakrabartty, S. N. (2019). Limitations of Insomnia Severity Index and possible remedies, *JSM Neurological Disorders and Stroke*, 5(9), 1 – 9

Clench-Aas, J., Nes, R. B., Dalgard, O. S., & Aarø, L. E. (2011). Dimensionality and measurement invariance in the Satisfaction with Life Scale in Norway. *Quality of Life Research*, 20(8), 1307-1317. https://doi.org/10.1007/s11136-011-9859-x

Cortina J. (1993). What is coefficient alpha: an examination of theory and applications. *Journal of applied psychology*; 78: 98-104. https://doi.org/fn7g4t

Diener E., & Chan M. Y. (2011). Happy people live longer: Subjective well-being contributes to health and longevity. *Appiedl Psychology*; Health and Well-Being; 3 (1):1–43. DOI: https://doi.org/10.1111/j.1758-0854.2010.01045.x

Diener, E., Emmons, R.A., Larson, R. J., & Griffin, S. (1985). The satisfaction with life scale. *Journal of Personality Assessment*, 49, 71-75. https://doi.org/10.1207/s15327752jpa4901_13

Diener, E., & Emmons, R. A. (1984). The independence of positive and negative affect. *Journal of Personality and Social Psychology*, 47, 1105–1117

Ehrhardt, JJ; Saris, W. E., & Veenhoven, R. (2000). Stability of Life-satisfaction Over Time: Analysis of change in ranks in a national population *Journal of Happiness Studies, vol. 1, 177-205*

Eid, M., Langeheine, R., & Diener, E. (2003): Comparing typological structures across cultures by multigroup latent class analysis: A primer. *Jr. of Cross-Cultural Psychology*, 34, 195-210

Emerson, S. D., Guhn, M., & Gadermann, A. M. (2017). Measurement invariance of the Satisfaction with Life Scale: reviewing three decades of research. *Qual Life Res* 26, 2251–2264 https://doi.org/10.1007/s11136-017-1552-2

Galanakis, M., Lakioti, A., Pezirkianidis, C., Karakasidou, E., & Stalikas, A. (2017). Reliability and Validity of the Satisfaction with Life Scale (SWLS) in a Greek Sample, *The International Journal of Humanities & Social Studies,* 5 (2), 120-127

Huebner, E. S. (1991). Initial development of the student's life satisfaction scale. *School Psychology International, 12*(3), 231-240. https://doi.org/b8hrb7

Jang, S., Kim, E. S., Cao, C., Allen, T. D., Cooper, C. L., Lapierre, L. M., O'Driscoll, M. P., Sanchez, J. I., Spector, P. E., Poelmans, S. A. Y., Abarca, N., Alexandrova, M., Antoniou, A.-S., Beham, B., Brough, P., Carikci, I., Ferreiro, P., Fraile, G., Geurts, S., … Woo, J.-M. (2017). Measurement Invariance of the Satisfaction With Life Scale Across 26 Countries. *Journal of Cross-Cultural Psychology, 48*(4), 560–576. https://doi.org/10.1177/0022022117697844

Karakasidou, E., Pezirkianidis, C., Stalikas, A., & Galanakis, M. (2016). Standardization of the Subjective Happiness Scale (SHS) in a Greek Sample. *Psychology*, 7, 1753-1765. http://dx.doi.org/10.4236/psych.2016.714164

Kern, J. L., McBride, B. A., Laxman, D. J., Dyer, W. J., Santos, R. M., & Jeans, L. M. (2016). The role of multiple-group measurement invariance in family psychology Research. *Journal of Family Psychology, 30*(3), 364-374. https://doi.org/10.1037/fam0000184

Kulczycka, L., Sysa-Jędrzejowska, A., & Robak, E. (2010). Quality of life and satisfaction with life in SLE patients-the importance of clinical manifestations, *Clinical Rheumatology*, *29*(9), 991–997

Laranjeira, C. A. (2009). Preliminary validation study of the Portuguese version of the satisfaction with life scale, *Psychology, Health and Medicine*, 14(2), 220–226

Livingston SA. (2004). *Equating test scores (without IRT*). Princeton, NJ: ETS

López-Ortega, M., Torres-Castro, S., & Rosas-Carrasco, O. (2016). Psychometric properties of the Satisfaction with Life Scale (SWLS): secondary analysis of the Mexican Health and Aging Study. *Health Qual Life Outcomes* 14, 170. https://doi.org/10.1186/s12955-016-0573-9

Lubin, B., & Van Whitlock, R. (2004). Psychometric properties of the brief life satisfaction scales. *Journal of clinical psychology*, *60*(1), 11-27. https://doi.org/10.1002/jclp.10190

Lubke, G. H., Dolan, C. V., Henk Kelderman, H., & Mellenbergh, G. J. (2003). On the relationship between sources of within- and between-group differences and measurement invariance in the common factor model, *Intelligence, 31* (6), 543-566. https://doi.org/10.1016/S0160-2896(03)00051-5.

Lucas-Carrasco, R., Den Oudsten, B. L., Eser, E., & Power, M. J. (2014). Using the satisfaction with life scale in people with Parkinson's disease: a validation study in different european countries. *The Scientific World Journal*, 2014, Article ID 680659. https://doi.org/10.1155/2014/680659

Mahmoud, J. S. R., Staten, R. T., Hall, L. A., & Lennie, T. A. (2012). The relationship among young adult college students' depression, anxiety, stress, demographics, life satisfaction, and coping styles. *Issues in Mental Health Nursing*, *33*(3), 149- 156.

Margolis, S., Schwitzgebel, E., Ozer, D.J., & Lyubomirsky, S. (2018). A new measure of life satisfaction: the riverside life satisfaction scale. *Journal of Personality Assessment*. 101(6), 621-630. https://doi.org/d5pc

Meadow, H.L., Mentzer, J.T., Rahtz, D.R. et al. A life satisfaction measure based on judgment theory. *Soc Indic Res* 26, 23–59 (1992). https://doi.org/fjkrmn

Meyer, C., Rumpf, H-J., Hapke, U., & John, U. (2004). Impact of psychiatric disorders in the general population: satisfaction with life and the influence of comorbidity and disorder duration, *Social Psychiatry and Psychiatric Epidemiology*, *39*(6), 435-441

Oishi, S. (2006). The concept of life satisfaction across cultures: An IRT analysis. *Journal of Research in Personality,* 40, 411-423.

Pavot, W., Diener, E., & Suh, E. (1998). The temporal satisfaction with life scale. Journal of personality Assessment, 70(2), 340-354. https://doi.org/d87dn8

Pavot, W., & Diener, E. (2013). The Satisfaction with Life Scale (SWL), Measurement Instrument Database for the Social Science. http://dx.doi.org/10.13072/midss.467

Pavot, W., & Diener, E. (1993). Review of the Satisfaction with Life Scale. *Psychological Assessment*, 5, 164-172.

Pavot, W., Diener, E. D., Colvin, C. R., & Sandvik, E. (1991). Further validation of the Satisfaction with Life Scale: Evidence for the cross-method convergence of well-being measures. *Journal of Personality Assessment*, 57(1), 149-161.

Schutte, L., Negri, L., Delle Fave, A. & Wissing, M.P. (2019). Rasch analysis of the Satisfaction with Life Scale across countries: Findings from South Africa and Italy. *Current Psychology*. https://doi.org/10.1007/s12144-019-00424-5

Shin, D. C., & Johnson, D. M. (1978). Avowed happiness as an overall assessment of the quality of life. *Social Indicators Research*, 5, 475–492. https://doi.org/dw5ddp

Steger, M. F., Frazier, P., Oishi, S., & Kaler, M. (2006). The meaning in life questionnaire: Assessing the presence of and search for meaning in life. *Journal of Counseling Psychology, 53*(1), 80–93. https://doi.org/c9kmrc

Tate, R. L. (2010). *A compendium of tests, scales, and questionnaires: The practitioners guide to measuring outcomes after acquired brain impairment*. NY, Psychology Press. ISBN 978-1-84169-561-7

Van Beuningen, J. (2012). *The Satisfaction with Life Scale examining construct validity*. Statistics Netherlands, The Hague/Heerlen, ISSN: 1572-0314

Van De Schoot R., Schmidt P., De Beuckelaer A., Lek K., & Zondervan-Zwijnenburg M. (2015). Editorial: Measurement Invariance. *Front Psychol*, 6. https://doi.org/10.3389/fpsyg.2015.01064

Vazquez, C., Duque, A., & Hervas, G. (2013). Satisfaction with life scale in a representative sample of Spanish adults: validation and normative data. *Spanish Journal of Psychology*, *16*(82), 1-15. https://doi.org/10.1017/sjp.2013.82

Vera-Villarroel, P., Urzúa, A., Celis-Atenas, P. P. K., & Silva, J. (2012). Evaluation of subjective well-being: Analysis of the satisfaction with life scale in Chilean population. *Universitas Psychologica*, *11*(3), 719-727. http://bit.ly/3FXu0Y2

Vittersø, J., Biswas-Diener, R., & Diener, E. (2005). The divergent meanings of life satisfaction: Item response modeling of the Satisfaction with Life Scale in Greenland and Norway. *Social Indicators Research*, *74*(2), 327-348, https://doi.org/10.1007/s11205-004-4644-7

Welzel, C., Kruse, S., & Brunkert, L. (2022). Against the Mainstream: On the Limitations of Non-Invariance Diagnostics: Response to Fischer et al. and Meuleman et al. *Sociological Methods & Research*, 1-18. https://doi.org/10.1177/00491241221091754