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Python for financial engineers: Mastering four moments in portfolio management.

Python para engenheiros financeiros: Dominar quatro momentos da gestão de carteiras

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Abstract: This research conducts a comprehensive analysis aimed at optimizing portfolios comprising 14 stocks listed on the Moroccan stock exchange. Our journey culminates in the construction of portfolios that are meticulously designed to maximize returns while prudently managing risk. These portfolios are the result of an exhaustive Monte Carlo simulation that explored over three million unique portfolio combinations. The simulations take into account the skewness and kurtosis of the return distributions, offering investors a robust framework for decision-making. We collected historical data for theses 14 stocks on the Moroccan market exchange by accessing 5 years' worth of historical data from *investing.com*.We explore the concepts of Modern Portfolio Theory (MPT), which forms the backbone of our approach, and we employ the power of mathematics and Python programming to bring forth insights that can inform sound investment decisions. The primary focus of this study centers on the incorporation of higher statistical moments from the returns of key financial indices, with a particular emphasis on their skewness and kurtosis characteristics. To achieve this goal, various evaluative criteria derived from these statistical parameters are introduced and thoroughly investigated. Within this research framework, we confront a spectrum of optimization challenges, including the maximization of skewness, and minimization of kurtosis.

Keywords: Assets, Investment, Portfolio Theory, Python programming, Digital management

Resumo: Este estudo efetua uma análise exaustiva com vista a otimizar carteiras compostas por 14 ações cotadas na bolsa marroquina. O nosso percurso culmina com a construção de carteiras meticulosamente concebidas para maximizar os rendimentos e gerir prudentemente o risco. Estas carteiras são o resultado de uma simulação exaustiva de Monte Carlo que explorou mais de três milhões de combinações únicas de carteiras. As simulações têm em conta a assimetria e a curtose das distribuições de rendibilidade, oferecendo aos investidores um quadro robusto para a tomada de decisões. Recolhemos dados históricos para estas 14 ações na bolsa de valores marroquina, acedendo a 5 anos de dados históricos da investing.com. Exploramos os conceitos da Teoria Moderna da Carteira (MPT), que constitui a espinha dorsal da nossa abordagem, e empregamos o poder da matemática e da programação *Python* para obter conhecimentos que podem informar decisões de investimento sólidas. O foco principal deste estudo centra-se na incorporação de momentos estatísticos mais elevados dos retornos dos principais índices financeiros, com particular ênfase nas suas características de assimetria e curtose. Para atingir este objetivo, são introduzidos e investigados vários critérios de avaliação derivados destes parâmetros estatísticos. Dentro deste quadro de investigação, confrontamos um espetro de desafios de otimização, incluindo a maximização da assimetria e a minimização da curtose.

Palavras-chave: Ativos, Investimento, Teoria da carteira, Programação Python, Gestão digital

1. Introduction:

In the realm of modern portfolio theory, the mean-variance model, first introduced by Markowitz, has emerged as a pivotal and widely accepted tool for optimizing portfolios. Markowitz's pioneering framework has fundamentally reshaped perceptions of asset portfolios, leading to a proliferation of research on portfolio selection primarily grounded in the first two moments of return distributions. Nevertheless, an ongoing debate persists regarding whether the incorporation of higher moments should be a factor in the process of portfolio selection. (See Samuelson, 1970; Arditti and Levy 1975; Kraus and Litzenberger, 1976; Singleton and Wingender,1986; Prakash et al., 2003, and Sun and Yan 2003).

Engaging higher moments for portfolio optimization introduces a heightened level of complexity compared to standard methods like the mean-variance approach. This complexity stems from the delicate balance between conflicting objectives; namely, investors strive to maximize expected returns and skewness (Arditti, F. D. (1975)), all the while minimizing variance and kurtosis (See Gonçalves, G. W. (2022). To tackle this complex portfolio challenge involving multiple goals, we need to employ certain numerical methods, as often there's no straightforward answer available. Extending the work of Lai et al. (2006), this study employs Polynomial Goal Programming (PGP) as an advanced approach, which effectively integrates investors' preferences pertaining to higher statistical moments into the optimization process. Notable authors in the field, such as Wanke, P. F., Tan, Yong, and Gonçalves, Guilherme have advocated for the comprehensive analysis of risk, highlighting the importance of understanding the asymmetric nature of returns and the potential for extreme events. Recent research in the field, published in [ELSEVIER, A higher order portfolio optimization model incorporating information entropy, 2022] underscores the significance of embracing these higher moments in portfolio construction.

In today's dynamic and ever-evolving financial landscape, the art of investment management is undergoing a transformative shift. Modern investors are not just seeking returns; they are navigating an intricate terrain of risk, volatility, and opportunity.

In this research work, our exploration centers around a diverse set of fourteen Moroccan stocks, each with its unique story and potential. These equities, ranging from Agma to Label Vie, represent a cross-section of industries and economic sectors within Morocco. Their interactions within the market provide a rich tapestry of opportunities and challenges for investors, whether they are individual traders or institutional giants. The Moroccan stock market, much like its global counterparts, operates on the fundamental principles of risk and reward. Investors aim to balance the pursuit of maximizing returns while vigilantly safeguarding their investments. Traditionally, the investment world has relied heavily on metrics such as mean returns and standard deviations, crafting portfolios based on volatility and correlation. However, the landscape has evolved, and in our quest for deeper insights, we now venture into the realm of higher moments - skewness and kurtosis.

To achieve this objective, we employ the principles of Modern Portfolio Theory (MPT), developed by Nobel laureate Harry Markowitz, which forms the backbone of our approach. By harnessing the power of mathematics and advanced computational tools, we navigate the complex terrain of portfolio optimization.

Our journey culminates in the construction of portfolios that are meticulously designed to maximize returns while prudently managing risk. These portfolios are the result of an exhaustive Monte Carlo simulation (Rubinstein, M. &. (2016)) that explored over three million unique portfolio combinations. The simulations take into account the skewness and kurtosis of the return distributions, offering investors a robust framework for decision-making.

Our aspiration is to empower investors with a deeper understanding of portfolio diversification, emphasizing the significance of skewness and kurtosis alongside traditional risk and return metrics. By doing so, we provide the tools and insights needed to make informed investment decisions in the Moroccan stock market, enhancing the probability of profitable outcomes and managing the impact of extreme events - both positive and negative.

As we navigate this case study, we will explore the multifaceted world of portfolio management and discover how the integration of higher moments can redefine investment strategies in a dynamic and ever-changing financial landscape.

2. Materiel and methods:

2.1 Materiel

In the vibrant realm of the Moroccan Stock Market Exchange, fourteen prominent stocks have taken center stage, each with its own unique story to tell. These equities - Agma, Attijariwafa Bank, Afriquia Gaz, Managem, Disway, Total Maroc, Maghreb Oxygene, Atlanta, Societe de Therapeutique Marocaine S.A, Mutandis, Sonasid, Ennakl, LafargeHolcim, and Label Vie - represent a diverse cross-section of industries and economic sectors. They are the building blocks of financial aspirations, both for individual investors and institutional giants. Portfolios considered in this study are the result of an exhaustive ***Monte Carlo simulation*** that explored ***over three million unique portfolio combinations.*** We collected historical data for theses 14 stocks on the Moroccan market exchange by accessing ***5 years' worth of historical data*** from ***investing.com.***

The Moroccan stock market, like many others around the world, thrives on the principles of risk and reward. It is a bustling arena where investors seek to strike a balance between maximizing returns and safeguarding their investments.

Traditionally, the investment world has focused on the first two moments of the return distribution - the mean (return) and the standard deviation (risk), meticulously constructing portfolios based on volatility and correlation. However, recognizing the intricacies of asset behavior and the impact of extreme events, we step beyond the confines of volatility. In our quest for deeper insights, we turn to higher moments - skewness and kurtosis. Skewness offers a lens into the asymmetric nature of returns, while kurtosis unveils the degree of tail risk, highlighting the potential for extreme outcomes.

We explore the concepts of Modern Portfolio Theory (MPT), which forms the backbone of our approach, and we employ the power of mathematics and ***Python programming*** to bring forth insights that can inform sound investment decisions.

2.2. Methods: Mathematical approach for Skewness and Kurtosis in Portfolio Management

Investing in financial markets is a complex endeavor, and every investor aims to achieve a balance between maximizing returns and managing risk (Choueifaty & Coignard). In the world of finance, one groundbreaking concept has played a pivotal role in shaping the way investors approach portfolio management - the Mean-Variance Model. This model is not just a set of mathematical equations but a powerful framework that encapsulates how investors think about constructing portfolios and why it is beneficial to their financial success (See Prakash, A. J. (2003), Rachev, S. T. (2012),).

At its core, the Mean-Variance Model is rooted in the idea that investors are inherently risk-averse. In other words, they prefer to avoid excessive risk when making investment decisions . However, they also seek to maximize their potential returns. This inherent dilemma, balancing risk and return, is at the heart of every investor's decision-making process. Here's where the Mean-Variance Model steps in as a guiding light.

This model is beneficial to investors because it offers a systematic approach to navigate this intricate trade-off. It provides a structured way of thinking about how to allocate capital across different assets or investments to achieve an optimal mix. In simple terms, it helps investors answer questions like: How much of my portfolio should I allocate to stocks, bonds, real estate, or other assets? What's the right balance between potential returns and the risk I'm willing to accept?

By addressing these questions, the Mean-Variance Model assists investors in constructing portfolios that align with their risk tolerance, financial goals, and investment horizon. It isn't about avoiding risk altogether; instead, it's about understanding and quantifying the risk-return relationship. This model allows investors to make informed decisions by striking a balance between aiming for higher returns and mitigating excessive risk.

2.2.1 The key components of the Mean-Variance Model:

- **Expected Return (Mean):** This represents the average return an investor can anticipate from an investment or a portfolio. It is a measure of the potential gain.
- **Variance (Risk):** Variance measures the dispersion of returns around the expected return. A lower variance indicates lower risk, while a higher variance implies higher risk.
- **Covariance:** Covariance measures how two assets move in relation to each other. A positive covariance means the assets tend to move in the same direction, while a negative covariance suggests they move in opposite directions.
- **Portfolio Diversification:** The Mean-Variance Model encourages diversification, which means investing in a mix of assets with low or negative correlations to spread risk.

- **Efficient Frontier:** The efficient frontier is a graphical representation of portfolios that offer the highest expected return for a given level of risk.

2.2.2 / The third (Skewness) and fourth moment (Kurtosis):

Skewness (S) and kurtosis (K) are higher moment statistics that describe the shape and tail behavior of the return distribution. In mathematical terms:

a/ Simplified Skewness Formula (Univariate Skewness):

The simplified formula for skewness is used to measure the skewness of a single-variable distribution (univariate skewness). It quantifies the asymmetry of the distribution of a single random variable.

The formula for univariate skewness is typically expressed as:

$$S_{simplified} = \frac{E[(X - \mu)^3]}{\sigma^3}$$

Where X is the portfolio return, μ is the mean of the distribution, σ is the standard deviation, and E denotes the expected value.

This formula calculates how much the distribution of a single variable deviates from perfect symmetry. Positive skewness indicates a longer right tail (right-skewed), while negative skewness indicates a longer left tail (left-skewed).

b/ Complex Skewness Formula (Portfolio Skewness):

The more complex formula for skewness is used when dealing with portfolios containing multiple assets. It measures the skewness of the portfolio's returns, taking into account both the individual asset returns and their interdependencies. The formula for portfolio skewness can be expressed as follows:

$$S_{complex} = \frac{N}{(N-1)(N-2)} \sum_{i=1}^{N} w_i \left(\frac{X_i - \mu}{\sigma}\right)^3$$

This formula calculates the skewness of a portfolio by considering the weighted sum of the third moments (skewness) of each asset's returns, normalized by the portfolio's standard deviation.

The key difference is that the simplified skewness formula is for a single variable, measuring the skewness of its distribution, while the complex skewness formula is for a portfolio with multiple assets, considering both the individual asset returns and their interdependencies to measure the skewness of the portfolio's returns.

The simplified formula for kurtosis for a univariate distribution (single-variable distribution) is as follows:

$$K_{simplified} = \frac{E[(X-\mu)^4]}{\sigma^4}$$

Where X is the portfolio return, μ is the mean of the distribution, σ is the standard deviation, and E denotes the expected value. This formula measures the kurtosis of a single variable.

Kurtosis containing n stocks can be expressed mathematically using the following formula, where X_i represents the return of the ith stock:

Let $X_1, X_2, X_3, \dots, X_n$ represent returns of the n stocks in the portfolio. The kurtosis ($K_{complex}$) of the portfolio's returns can be calculated as:

$$K_{complex} = \frac{N(N+1)}{(N-1)(N-2)(N-3)} \sum_{i=1}^{N} w_i \left(\frac{X_i-\mu}{\sigma}\right)^4 - \frac{3(N-1)^2}{(N-2)(N-3)}$$

This formula calculates the excess kurtosis (i.e., kurtosis relative to a normal distribution) of the portfolio's returns by considering the kurtosis of each individual stock's returns (X_i) and their respective weights(w_i). The formula normalizes the kurtosis values for each stock by dividing them by the standard deviation (σ) of the portfolio's returns.

The term $\frac{3(N-1)^2}{(N-2)(N-3)}$ is subtracted to adjust for the excess kurtosis. Positive excess kurtosis indicates heavier tails (more outliers) compared to a normal distribution, while negative excess kurtosis indicates lighter tails.

2.2.3 / Mathematical formulas for portfolio optimization objectives:

2.2.3.1 / Maximizing Portfolio Skewness:

Skewness measures the asymmetry of returns. A portfolio with a high skewness indicates that it has the potential for significant positive returns, even though it may have some negative returns. Maximizing skewness can be appealing to investors looking for portfolios with the potential for substantial upside gains.

Objective: $$Maximize\ S_{simplified} = \frac{E[(X-\mu)^3]}{\sigma^3}$$

$$Maximize\ S_{complex} = \frac{N}{(N-1)(N-2)}\sum_{i=1}^{N} w_i \left(\frac{X_i-\mu}{\sigma}\right)^3$$

$$Subject\ to: \left\{\sum_{i=1}^{N} w_i = 1\ (weight\ constraint)\right.$$

2.2.3.2 / Minimizing Portfolio Kurtosis:

Kurtosis measures the ‘tailedness’ of returns. By minimizing kurtosis, we aim to identify portfolios with returns that are less extreme and more closely aligned with a normal distribution. This can be suitable for investors who prefer smoother, less volatile returns.

Objective: $$Minimize\ K_{simplified} = \frac{E[(X-\mu)^4]}{\sigma^4} - 3$$

$$Minimize\ K_{complex} = \frac{N(N+1)}{(N-1)(N-2)(N-3)}\sum_{i=1}^{N} w_i \left(\frac{X_i-\mu}{\sigma}\right)^4 - \frac{3(N-1)^2}{(N-2)(N-3)}$$

$$Subject\ to: \left\{\sum_{i=1}^{N} w_i = 1\ (weight\ constraint)\right.$$

By identifying these key portfolios, investors gain access to a spectrum of options that cater to their specific risk return skewness kurtosis preferences. The exact mathematical formulation for each portfolio involves quadratic optimization problems with constraints on asset weights. These constraints are essential to ensure that the portfolio is practical and adheres to predefined investment guidelines.

3. Result and discussion

In order to construct portfolios that maximize returns while intelligently managing risk, we seek to mitigate excessive risk exposure while comprehensively analyzing higher moments. By closely examining skewness and kurtosis, we aim to gain a deeper understanding of volatility—how these stocks oscillate in the market, and how they handle extreme events. Our aspiration is to craft portfolios that are not only risk-aware but also strategically positioned to harness the power of positive skewness, thereby increasing the probability of profitable outcomes, all the while exploring the tails of the return distribution, providing insights into the likelihood of extreme events - both positive and negative.

These portfolios, created through a meticulous Monte Carlo simulation in Python, represent the outcome of exploring over three million portfolio combinations. They are the result of data-driven decision-making, designed to align with a wide array of investment objectives and risk tolerances.

Table 1. Performance Metrics of Diversified Portfolios

Portfolios	Return	Risk	Sharpe	Skewness	Kurtosis
Highest return	16.10%	13.10%	0.907	0.545	1.081
Lowest risk	10.80%	10.00%	0.662	-0.027	0.1965
Highest Sharpe	14.60%	10.90%	0.955	-0.005	0.297
Highest Skewness	11.80%	14.30%	0.530	2.412	12.634
Lowest Kurtosis	12.50%	11.00%	0.750	0.078	-0.725

In the table provided, we have a comprehensive breakdown of five distinct portfolios, each optimized to meet specific investment objectives. Let's delve into the details:

a/ Portfolio with the Highest Return:

This portfolio is designed for investors with an appetite for maximizing returns. It has delivered an impressive annual return of 16.10%. However, it comes with a moderate level of risk, as indicated by a 13.10% standard deviation. The Sharpe ratio, a measure of risk-adjusted return, stands at a strong 0.907. Furthermore, this portfolio exhibits positive skewness (0.545) and kurtosis (1.081), suggesting a somewhat right-skewed return distribution with a slightly fat tail.

- Return: With an impressive 16.10% annual return, this portfolio has shown its potential for delivering substantial gains.
- Risk: The 13.10% standard deviation reflects a moderate level of risk, indicating some fluctuations in returns.
- Sharpe Ratio (0.907): This ratio suggests that for each unit of risk undertaken, the portfolio has historically delivered a strong risk-adjusted return.
- Skewness (0.545): Positive skewness hints at the potential for larger upside returns, although it's not excessively skewed.
- Kurtosis (1.081): The kurtosis value indicates that the portfolio's return distribution has slightly heavier tails than a normal distribution, which implies a small probability of extreme returns.

b/ Portfolio with the Lowest Risk:

For those prioritizing capital preservation, the lowest risk portfolio offers an attractive option. It has achieved an annual return of 10.80% with a remarkably low risk level of 10.00%. The Sharpe ratio of 0.662 indicates a good risk-adjusted performance. Additionally, this portfolio demonstrates a slightly negatively skewed return distribution (-0.027) and relatively low kurtosis (0.1965), signifying a more stable and less volatile investment profile.

- Return: Despite its focus on minimizing risk, this portfolio has still managed a respectable annual return of 10.80%.
- Risk: The exceptionally low 10.00% standard deviation demonstrates a commitment to capital preservation.
- Sharpe Ratio (0.662): While the Sharpe ratio is lower compared to other portfolios, it highlights the solid risk-adjusted performance given its minimal risk exposure.

- Skewness (-0.027): Slightly negative skewness suggests a mildly left-skewed return distribution, indicating a tendency for smaller downside fluctuations.
- Kurtosis (0.1965): The kurtosis value signifies a relatively thin tail, signifying a more stable investment profile.

c/ Portfolio with the Highest Sharpe Ratio:

Investors aiming for the optimal balance between risk and return will appreciate this portfolio. With an annual return of 14.60% and a risk level of 10.90%, it boasts a Sharpe ratio of 0.955, reflecting its superior risk-adjusted performance. The skewness is close to zero (-0.005), suggesting a near-normal distribution, while the kurtosis is 0.297, indicating a relatively thinner tail.

- Return: With a 14.60% annual return, this portfolio balances risk and reward effectively.
- Risk: The 10.90% standard deviation showcases a prudent level of risk.
- Sharpe Ratio (0.955): An impressive Sharpe ratio suggests that this portfolio has historically excelled in providing risk-adjusted returns.
- Skewness (-0.005): Near-zero skewness indicates a return distribution close to normal, with well-balanced upside and downside potential.
- Kurtosis (0.297): A kurtosis value close to zero implies a thinner tail and less extreme returns.

d/ Portfolio with the Highest Skewness:

This portfolio caters to those seeking potentially higher returns with an elevated level of risk. It has delivered an annual return of 11.80%, albeit with a higher risk level of 14.30%. The Sharpe ratio of 0.530 reflects a lower risk-adjusted return. Notably, this portfolio exhibits significant positive skewness (2.412), implying the potential for positively skewed returns, and a relatively high kurtosis (12.634), indicating a distribution with a heavy tail.

- Return: Despite its riskier profile, this portfolio has delivered an annual return of 11.80%.
- Risk: The 14.30% standard deviation indicates a higher level of risk compared to some other portfolios.

- Sharpe Ratio (0.530): The lower Sharpe ratio reflects the trade-off between potential higher returns and increased risk.
- Skewness (2.412): The substantial positive skewness suggests the potential for larger, positively skewed returns, albeit with a higher degree of risk.
- Kurtosis (12.634): The high kurtosis value points to a distribution with heavier tails, implying the possibility of extreme returns.

e/ Portfolio with the Lowest Kurtosis:

For investors who prefer a more predictable and less extreme return distribution, the lowest kurtosis portfolio is a suitable choice. It has generated an annual return of 12.50% with a risk level of 11.00%. The Sharpe ratio of 0.750 indicates a favorable risk-adjusted return. The skewness is positive (0.078), suggesting a modest right-skewed distribution, while the kurtosis is negative (-0.725), indicating a thinner tail compared to the norm.

- Return: This portfolio has achieved an annual return of 12.50% while prioritizing a more predictable distribution.
- Risk: The 11.00% standard deviation balances risk and return effectively. Sharpe Ratio (0.750): The favorable Sharpe ratio indicates solid risk-adjusted returns while avoiding extreme tail risk.
- Skewness (0.078): Positive skewness suggests a modest potential for upside returns.
- Kurtosis (-0.725): The negative kurtosis value highlights a thinner tail compared to a normal distribution, indicating lower risk of extreme returns.

Let’s discuss the portfolio allocations based on the provided weights. As an investor, it's essential to understand how these allocations impact the composition and potential performance of each portfolio:

Table 2. Portfolio Allocations: Weightings for Different Investment Objectives

Weights in %	AGM	ATW	GAZ	MNG	DWY	TMA	MOX	ATL	SOT	MUT	SID	NKL	HLM	LBV
Highest return	11.90	0.00	14.50	8.30	9.90	2.60	11.90	0.80	18.50	1.00	1.20	0.20	0.60	18.50
Lowest risk	16.50	0.80	5.10	0.30	12.10	14.90	3.90	0.20	6.30	16.10	0.00	11.50	2.50	9.70
Highest Sharpe	17.00	1.10	2.10	2.80	6.20	12.40	1.60	0.70	20.00	14.30	0.90	0.00	0.90	20.00
Highest Skew	17.00	4.10	2.20	2.50	1.60	9.90	26.30	15.30	5.50	4.90	1.10	7.70	1.40	0.5
Lowest Kurt	20.10	0.70	2.20	0.70	15.00	17.90	0.40	0.90	9.30	1.10	7.50	5.70	4.00	14.60

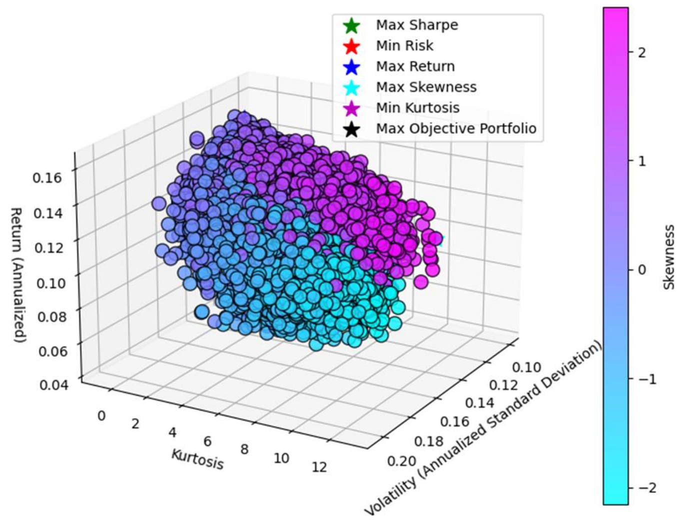
- ✓ ***Portfolio with the Highest Return***: This portfolio prioritizes maximizing returns, and it's evident in its weightings. Notably, it has a substantial allocation to 'ATL' (18.50%) and 'LBV' (18.50%), indicating a strong focus on these assets. The inclusion of 'AGM,' 'GAZ,' and 'TMA' also contributes significantly to potential returns. However, it's important to note that a diversified approach might be lacking, as some assets have minimal or zero weight.
- ✓ ***Portfolio with the Lowest Risk***: Investors seeking stability and capital preservation will find solace in this portfolio. It heavily favors 'AGM' (16.50%) and 'MUT' (16.10%), emphasizing their low volatility attributes. 'DWY,' 'NKL,' and 'HLM' also receive notable allocations, contributing to risk mitigation. This portfolio offers a well-rounded mix with a preference for low-risk assets.
- ✓ ***Portfolio with the Highest Sharpe Ratio***: Efficiency in risk-adjusted returns is the focus here. 'ATL' (20.00%) and 'MOX' (20.00%) dominate the allocation, reflecting their potential for balanced risk and return. 'SOT' (14.30%) and 'DWY' (12.40%) also play crucial roles in optimizing the Sharpe ratio. This portfolio seeks to maximize the return per unit of risk.
- ✓ ***Portfolio with the Highest Skewness***: For those willing to accept higher risk for the potential of positively skewed returns, this portfolio is intriguing. 'SOT' (26.30%) and 'MUT' (15.30%) receive substantial allocations, emphasizing their potential for non-normally distributed, positively skewed returns. However, the portfolio's composition leans heavily toward a few assets, potentially increasing concentration risk.
- ✓ ***Portfolio with the Lowest Kurtosis***: This portfolio aims to reduce the probability of extreme returns by favoring assets with thinner tails in their return distributions. 'SOT' (7.50%) and 'HLM' (5.70%) receive significant allocations, reflecting their

characteristics of a thinner-tailed distribution. 'AGM,' 'DWY,' and 'NKL' also play roles in achieving this objective. However, diversification may be sacrificed for this goal.

As an investor, it's essential to align your portfolio selection with your risk tolerance, return expectations, and overall investment strategy. The weights provided offer a glimpse into the trade-offs between various assets, risk levels, and potential returns. Balancing these factors is key to constructing a portfolio that meets your specific financial objectives and preferences.

In the world of portfolio analysis and decision-making, visualization is a powerful tool that aids us in understanding the complex relationships between various financial metrics. One common approach is to use 2D plots that showcase the risk-return trade-off, often employing a colormap to represent the Sharpe ratio. While this method offers valuable insights, it may not provide the full picture of our data. To truly grasp the intricacies of portfolio diversification, we've taken a step forward by employing a 3D plot that incorporates risk, return, kurtosis, and a colormap representing skewness.

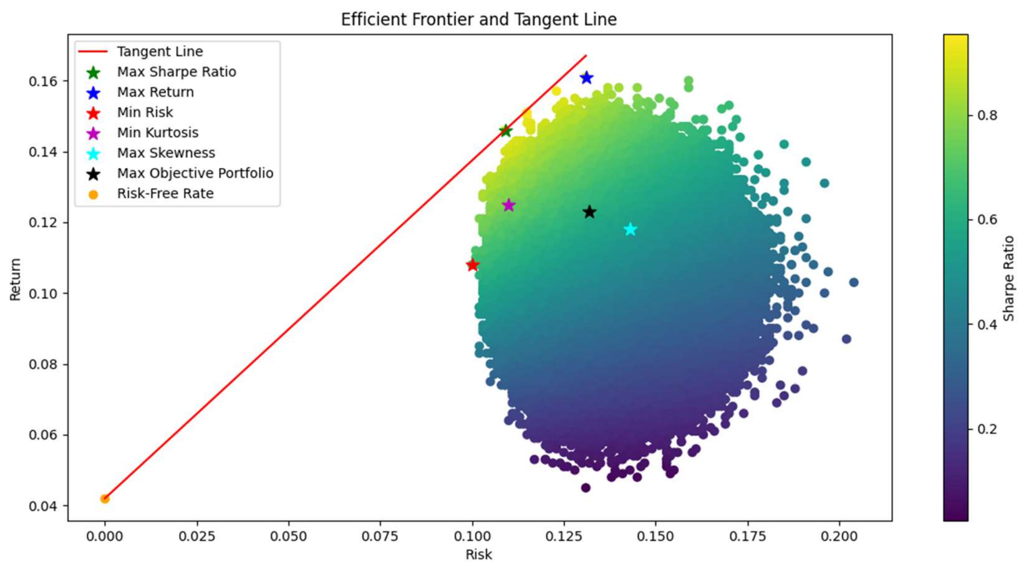
Figure 1. 3D plot incorporating risk, return, kurtosis, and a colormap representing skewness



By including skewness and kurtosis in our 3D plot, we gain a deeper understanding of the nuances in our portfolio. The colormap representing skewness adds a layer of complexity, helping us visualize the distribution of potential returns in a way that a simple Sharpe ratio cannot. Positive skewness suggests the potential for larger positive returns, while negative skewness hints at the possibility of larger negative returns. Kurtosis, on the other hand, informs us about the tails of the distribution. High kurtosis indicates the likelihood of extreme returns, be they exceptionally good or bad. This is particularly valuable because skewness can significantly impact the nature of returns, especially in cases where we are looking for investments with a particular risk profile.

In this next analysis, we present three separate 2D plots, each with its unique colormap, to offer a comprehensive view of our portfolio options. What sets these plots apart are the portfolios we've carefully marked with stars, including those designed for maximum return, maximum Sharpe ratio, maximum skewness, minimum kurtosis, and minimum risk. Let's delve into each plot:

Figure 2. 2D Risk-Return with Sharpe Ratio Colormap:

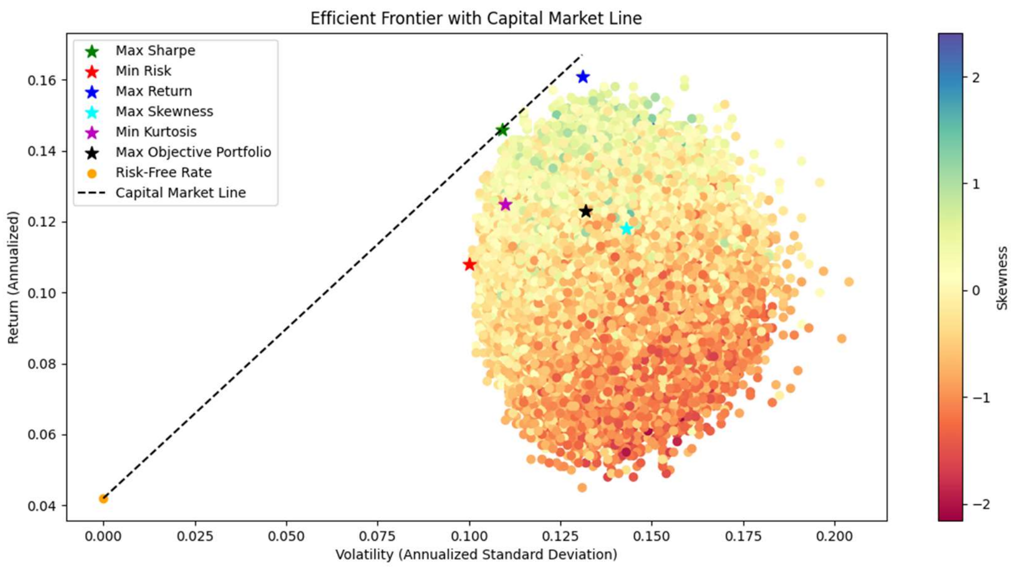


Maximum Return (★): This portfolio, marked with a star, is all about seeking the highest possible returns. It stands out with a substantial return, but it's essential to consider its associated risk level and Sharpe ratio for risk-adjusted performance.

Maximum Sharpe Ratio (★): This star indicates the portfolio optimized for efficient risk-adjusted returns. It balances risk and return optimally, reflecting a competitive Sharpe ratio.

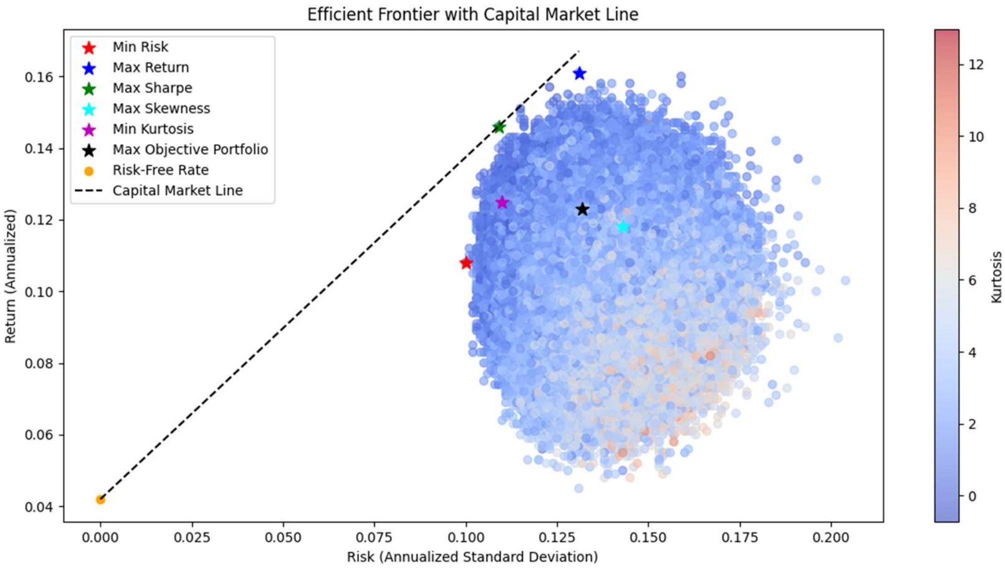
Other Portfolios: While not marked with stars, the plot displays a range of portfolios, each with its unique risk-return profile, helping investors visualize the trade-offs between risk and reward.:

Figure 3. 2D Risk-Return with Skewness Colormap



Maximum Skewness (★): Positively skewed returns are the hallmark of this portfolio. The star signifies its potential for larger upside gains. However, remember that higher skewness often comes with increased risk. This plot showcases a spectrum of portfolios with varying skewness levels. Some may exhibit positively skewed returns (right-leaning), while others might have negatively skewed returns (left-leaning).

Figure 4. 2D Risk-Return with Kurtosis Colormap



Minimum Kurtosis (★): Designed to reduce the likelihood of extreme returns, this portfolio's star emphasizes its thinner-tailed return distribution. While it might provide a more stable investment experience, it may also limit potential for exceptional gains.

Other Portfolios: The plot illustrates a range of portfolios, each with its kurtosis value, helping investors identify those with fatter or thinner tails in their return distributions.

These three plots serve as invaluable tools for portfolio analysis and decision-making. By visualizing risk and return in combination with Sharpe ratio, skewness, and kurtosis, investors gain a deeper understanding of the trade-offs and characteristics of various portfolios. The starred portfolios provide clear reference points for those pursuing specific investment objectives, whether it's maximizing returns, achieving efficient risk-adjusted performance, targeting skewed returns, or minimizing the risk of extreme outcomes.

Remember that the choice of portfolio is a multifaceted decision that should encompass both quantitative analysis and qualitative factors in building a well-balanced investment strategy.

4. Conclusion

In this case study, we embarked on a comprehensive analysis of 14 Moroccan stocks with the goal of constructing portfolios tailored to various investment objectives and risk preferences. These portfolios, characterized by their unique combinations of risk, return, Sharpe ratio, skewness, and kurtosis, offer valuable insights for navigating the complexities of the Moroccan stock market. It's worth noting that all the analytical work conducted in this study was programmed and executed using Python.

We identified five key portfolios optimized for specific financial goals, such as maximizing returns, minimizing risk, achieving an efficient risk-reward balance, pursuing positively skewed returns, and targeting a narrower distribution of returns. Visual representations in 2D plots, utilizing colormaps, enabled investors to compare trade-offs between risk and return across these portfolios, providing a clearer perspective on investment choices.

Furthermore, we introduced a 3D plot that integrated risk, return, kurtosis, and skewness, offering a more comprehensive view of return distributions. This approach underscored the significance of skewness and kurtosis in making informed investment decisions and optimizing portfolio diversification. In conclusion, the portfolios constructed in this study, all meticulously analyzed and programmed in Python, equip investors with valuable tools to navigate the Moroccan stock market's opportunities and challenges. These portfolios align with diverse investment goals and risk appetites, facilitating the creation of well-informed and diversified investment strategies.

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